# Training the Doubtful and Timid[*]

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August 21, 2014

### Abstract

We analyze the effect of human capital on insurance within the firm. In our model human capital creates rents that are split between workers and firms. These rents are lost when employment at a particular firm is terminated, and thus create a (limited) commitment device over which a long-term contract can be agreed upon. We characterize the optimal long-term contract and study the impact that changes in human capital have on the risk-sharing between the worker and the firm. When the firm's commitment is not binding, human capital does not affect insurance within the firm as long as the firm does not face financing costs. A worker in our model faces wage risk not because of changes in her productivity, but due to industry-wide shocks. This feature allows us to generate empirical predictions linking industry characteristics to the characteristics of the long-term contract.

*"[...] the system under which the confident and venturesome "assume the risk" or "insure" the doubtful and timid by guaranteeing to the latter a specified income [...]"*

# 1 Introduction

The Knightian view of the firm as a system for insuring risk-averse workers is at the center of explanations for a wide variety of observations regarding labor income and equity risk. The basic idea is that the firm can provide workers with insurance against income risk because its owners are well diversified, or are less risk-averse, than workers. The idea has first been formalized in a literature following Khilstrom and Laffont (1979) and has been combined with implicit contract theory by Holmstrom (1983) and Harris and Holstrom (1982).[1]

In this paper, we analyze the effect of worker human capital on "insurance within the firm", i.e. the transfer of on-the-job wage risk from workers to firms' owners.[2] "Insurance within the firm" can occur in our model due to two of its features. First, we model human capital generating rents that are split between workers and firms. Second, the rents associated with human capital will get lost if employment relationships are terminated. As a consequence, employment relationships may continue even at times when they generate losses. The maximum loss that will be tolerated is the present value of any rents that would be lost if an employment relationship ends. The maximum loss determines the extent to which a worker can be insured against shocks to productivity, i.e. the extent to which the worker can receive a wage above productivity.

We analyze two types of wage contracting: short-term and long-term contracting. Under short-term contracting, wages are determined after a worker's productivity becomes known. Short-term contracting specifies the baseline exposure to wage risk that workers

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[1] Other early models are Baily (1974) and Azariadis (1975).

[2] "Insurance within the firm" is the title of a seminal empirical analysis by Guiso, Schivardi, and Pistaferri (2005).

have in the absence of any insurance within the firm. The exposure results from shocks to worker productivity shared by a group of firms which can benefit from a certain type human capital. We refer to these firms as an "industry" and model the productivity shocks as shocks to the price of the industry's output, which is the state variable of our model. The analysis of short-term contracting shows that, even without the objective of providing workers with insurance within the firm, it may happen that a worker earns a wage above her productivity.

Long-term contracting specifies state-specific wages before the state is known. We consider two types of long-term contracting regimes. Following Holmstrom (1983), we first assume that a worker cannot commit to continuing an employment relationship, but her employer can. In the second type of long-term contracting regime, we will assume that neither party to a long-term employment contract can commit to continuing an employment relationship. In both contracting regimes, we characterize a worker's exposure to wage risk under long-term contracting in terms of the worker's marginal rate of substitution between additional income in different states.[3]

The analysis of the first long-term contracting regime yields a benchmark case characterized by an intuitive irrelevance result: if employers can commit to employing workers at pre-specified wages, human capital may not matter for workers' exposure to wage risk. The irrelevance result is obtained under the assumption that it is costless to finance any losses due to paying a worker a wage above the profit she generates. We relax the assumption by allowing for a cost of financing losses, thus following the lead of Froot, Scharfstein and Stein (1993). Introducing the financing cost destroys the irrelevance result. The higher the cost, the more costly it is to reduce a worker's exposure to wage risk through long-term contracting if that requires paying the worker a wage above her marginal profit.

Our paper is related to the growing literature about risk-sharing between workers and firms. The closest work is Harris and Holstrom's (1982) seminal work, in which they show that workers will receive a wage that is constant but increases occasionally since the workers

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[3]This version of the paper, however, only includes the first type.

would otherwise quit. Berk, Stanton and Zechner (2010) build on Harris and Holstrom (1982) to show that the wage profile can be decreasing when firms can declare bankruptcy in order to break implicit contracts insuring workers against wage risk. In our model, workers may also make wage concessions in order to protect future rents that would be lost upon termination of employment relationships. However, our analysis differs since we consider the case in which firms cannot *at all* commit to providing workers with insurance against wage risk under an implicit contract. This feature of our analysis also distinguishes it from a recent analysis by Zhang (2014) who endogenizes the firm's investment in the worker's human capital in a framework like that of Berk, Stanton and Zechner (2010). Our work provides fresh empirical predictions about risk-sharing within the firm, and thus can complement existing work that studies it and its consequences (among others, Beaudry and Dinardo (1991), Guiso et. al. (2005), Comin et. al. (2009), Philippon et al (2012), and more recently, Ouimet et. al. (2014)).

The structure of the paper is as follows. In section 2 we describe the model. Section 3 derives the wage profile that a worker will earn under short-term contracting. The main results of this section are in Proposition 1 and and Corollary 1.2. Proposition 1 characterizes the wages earned by the worker and Corollary 1.2 establishes that an increase in the worker's human capital will increase her exposure to on-the-job wage risk. Section 4 defines an industry equilibrium, in which prices and the amount of training workers get are endogenous. Section 5 derives the optimal long-term contract. The main result under long-term contracting is Proposition 2, which characterizes the wages earned by the worker under a long-term contract with full employer commitment.

The paper is work-in-progress. For now, we define the equilibrium under which the worker's human capital is endogenous and results from firms' decisions to train their workers. The long-term contracts section expresses a basic result for long-term contracts, but the full implications derived from the industry equilibrium are left for a future draft.

# 2 Setup

In this section, we present our basic setup; any assumptions that we add in subsequent sections will be labelled separately.

We model a worker's exposure to output price shocks that hit the worker's employer and also affect the worker's outside options, i.e. the wages that the worker can earn elsewhere. We refer to the worker's current employer as the "incumbent". The worker can also work for other employers, but her human capital is only useful to firms whose output is sufficiently similar to that of the incumbent. We refer to the latter group of firms as an "industry", and assume that the industry contains at least one firm other than the incumbent. The incumbent and all other firms in the industry maximize expected profits, but the worker is risk averse. The worker's preferences are represented by a strictly concave money-metric utility function $u(\cdot)$. All players discount using a discount factor $\delta$.

The worker's human capital matters to the firms in the industry since it determines the worker's productivity. We assume that the worker's production capacity is one unit of output, but the production cost depends on the worker's human capital. Since there are no differences in terms of production capacity between the skilled worker and an unskilled worker, our model allows for the possibility that the skilled worker's human capital generates a riskless profit. Given a level of human capital $h$, the skilled worker's production cost is:

$$c(h) = \bar{c} - h, \tag{1}$$

where $\bar{c}$ is the production cost of a worker without any human capital.[4]

We refer to a worker without human capital as an "unskilled" worker and assume that such workers can always be hired for a wage of $\underline{w}$. Our focus will however be on a worker with human capital $h > 0$ representing special skills that are only useful in jobs inside the industry. Our analysis will characterize the effect of the worker's human capital on her exposure to risk. The effect will depend on two substantive assumptions. First, we

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[4]We use this particular functional form for the production cost for analytical tractability. Our qualitative results do not depend on this choice.

assume that the worker's special skills are of two types: industry-specific human capital, $\alpha h$, and firm-specific human capital $(1 - \alpha)h$, for $1 \geq \alpha \geq 0$. Second, we assume that the worker's human capital is perishable in that the worker looses skills if she does not use them to produce output. The two types of human capital differ in perishability. The firm-specific human capital perishes once the worker stops to produce output for the incumbent, while the industry-specific human capital perishes once no firm in the industry employs the worker to produce output.

The worker will be exposed to risk because the demand for the industry's output is uncertain. For now, it suffices to specify the demand uncertainty in terms of variation in the output price. We assume that there is a sequence of operating periods, and that output price follows a Markov process with two states.[5] In any given operating period, the output price is $\pi_z$, where $z \in \{1, 2\}$ denotes the state. We pick state 1 to be the high-price state: $\pi_1 > \pi_2$, and assume that $\pi_1 - \bar{c} \geq \underline{w}$. State $z$ occurs with probability $p_z$.

Each operating period has the following structure. First, the output price $\pi_z$ becomes public knowledge at the start of the period. Then, the incumbent and any other firm in the industry send the worker (possibly unacceptable) bids in order to hire the worker (for an exogenous number of hours) during the operating period. We refer to these bids as "wage offers". If the worker accepts any firm's wage offer, then the worker's output adds to that of the firm. Alternatively, the worker can choose an employer outside the industry. We assume that, outside the industry, the worker will earn the wage $\underline{w}$ introduced above.[6]

Throughout the analysis, we will assume that the worker has no bargaining power in negotiating with the incumbent.[7] That is, the incumbent will make take-it-or-leave-it

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[5]We focus on a model with two states for computational simplicity, but it is straightforward to extend the analysis.

[6]The wage $\underline{w}$ has been defined above as the wage of unskilled workers. By assuming that the same wage is paid to the skilled worker in a job outside the industry, we implicitly assume that the worker's human capital is fully industry-specific. It would be easy to extend the model in order to allow for more general human capital. The assumption that $\underline{w}$ is constant in all states implies that the dynamics of wages in the industry are idyiosincratic. In reality some of the variation in an industry's wages is due to systematic variation in aggregate wages, and thus our results should be interpreted as industry wage risk in addition to aggregate wage risk.

[7]As discussed in Manning (2005, page 4), there is a - largely unresolved - problem of indeterminacy regarding the division of rents in relationships of bilateral monopoly like the one between the worker and

wage offers. Moreover, we will use the following tie-breaking condition: when indifferent between two choices, all players break ties by making choices that preserve as much human capital as possible.

This concludes the basic setup of the model. We now solve for the wages that workers would receive in the absence of long-term contracts. Short-term contracts "frame" the long-term contracts, as they constitute the best alternative for workers and employers against which a long-term contract is signed. Thus, we need to solve the short-term problem before analyzing the long-term contract.

# 3    Short-term contracting

We start our analysis considering the case when no long-term contracts can be written between the worker and the employer. In this case employment and wage offers are made every period, without any explicit or implicit agreement regarding future periods. We will characterize the wages that a worker will earn in each state (Proposition 1), and will analyze the worker's exposure to wage risk.

From now on, we consider a specific operating period ("the current period") in which the output price is known to be $\pi_z$. We focus on the case in which, before the current period, the worker has already been employed by the incumbent,[8] and analyze the worker's alternatives relative to continued employment with the incumbent.

To start the analysis, we consider the options available to a worker who has received an offer. The worker has two alternatives: employment outside the industry and employment in the industry, but with a firm other than the incumbent. Employment in the industry will dominate employment outside the industry if the following participation constraint (PC) holds:

$$\hat{E}_z := u(\hat{w}_z) + \delta \hat{E} \geq u(\underline{w}) + \delta \underline{E}, \tag{2}$$

the incumbent in our model. We thus follow Manning (2005) and assume that wages are set by employers. Our results depend on the assumption that employers have at least *some* bargaining power.

[8]This case will turn out to be the only case in which the worker's human capital has not disappeared before the current period.

where $\hat{w}_z$ denotes the highest wage that the worker is offered by another firm in the industry, $\delta$ is a discount factor, and $\hat{E}$ and $\underline{E}$ denote the values of future employment opportunities of the worker. $\hat{E}$ is the value of future employment opportunities obtained if the worker maintains her industry-specific human capital by producing output for a firm in the industry during the current period. $\underline{E}$ is the value of employment opportunities obtained by the worker if she leaves the industry and thus looses her human capital. The PC (2) states that the worker will stay in the industry if the value of her industry-specific human capital exceeds the utility gained in the current period by leaving the industry for a wage of $\underline{w}$ rather than staying and earning a wage of $\hat{w}_z$: $\delta(\hat{E} - \underline{E}) \geq u(\underline{w}) - u(\hat{w}_z)$.

To determine whether condition (2) holds, we first consider the right-hand side of the condition. If the worker switches to a job outside the industry, her human capital is lost, and she will earn the wage $\underline{w}$ in all future periods. Thus,

$$\underline{E} := u(\underline{w}) \frac{1}{1 - \delta}, \tag{3}$$

and the PC (2) can be stated as follows:

$$\hat{E}_z := u(\hat{w}_z) + \delta \hat{E} \geq \underline{E}. \tag{2'}$$

We next formally define the value $\hat{E}$ of any future employment opportunities that the worker has conditional on maintaining her industry-specific human capital in the current period. The latter set of employment opportunities includes employment outside the industry in future periods, therefore $\hat{E} \geq \underline{E}$. If the worker chooses between employment in and outside the industry so as to maximize expected utility,

$$\hat{E} := p_1 \max[\hat{E}_1, \underline{E}] + (1 - p_1) \max[\hat{E}_2, \underline{E}]. \tag{4}$$

where $\hat{E}_z$ has been defined above and depends on the wage $\hat{w}_z$. If the wage $\hat{w}_z$ is sufficiently small, $\hat{E}_z < \underline{E}$, and the PC (2') will be violated.

To derive the wage $\hat{w}_z$ which appears in condition (3), we need to consider the behavior of firms in the industry other than the incumbent. Those firms will bid to hire the worker in order to benefit from her industry-specific human capital, which is equally valuable to all of them. As Bertrand competitors, they will offer the worker a wage which fully accounts for the productivity of her industry-specific human capital in the current period.[9] The worker will therefore be offered the following wage:

$$\hat{w}_z := \min[\pi_z, \underline{w} + \bar{c}] - c(\alpha h), \tag{5}$$

where $c(\alpha h)$ is the production cost at which the worker will be able to produce output based on her industry-specific human capital $\alpha h$. If $\pi_z < \underline{w} + \bar{c}$, then the wage (5) is the highest possible wage that a firm in the industry (other than the incumbent) can offer without making a loss in the current period. If $\pi_z > \underline{w} + \bar{c}$, the wage $\hat{w}_z = \underline{w} + \bar{c} - c(\alpha h)$ exceeds the wage $\underline{w}$ outside the industry by the entire reduction in production costs that the worker can achieve due to her industry-specific human capital, i.e. $\bar{c} - c(\alpha h)$. At this wage, any firms in the industry (other than the incumbent) will be indifferent between employing the worker and employing an unskilled worker at a wage $\underline{w}$ in order to produce output at a total cost of $\underline{w} + \bar{c}$.

The interaction between the worker's options and the wage $\hat{w}_z$ constrain the range of prices for which this wage is attractive to the worker. We refer to these prices as "critical prices". These critical prices are such that the price in the high state is sufficiently high so that firms can profit on that state while offering the worker a wage at least as high as her outside option. On the low-state, the price has to be sufficiently high so that firms can profit on that state while offering the worker what turns out to be a *lower* wage than her outside option. Lemma 1.1 formalizes the result.

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[9]Any higher wage offers could only be justified by expected rents to be earned in future periods. However, such rents cannot exist since there will be Bertrand competition for the worker also in future periods, as long as the worker has industry-specific human capital.

**Lemma 1.1:**[10] *(i) If the wage $\hat{w}_z$ satisfies the worker's PC (2') only in the high-price state $z = 1$, then the constraint will bind for $\hat{w}_1 = \underline{w}$.*

*(ii) For the wage $\hat{w}_z$ to satisfy the PC (2) in the low-price state $z = 2$, it is necessary that the PC is satisfied in the high-price state $z = 1$. If the wage $\hat{w}_z$ satisfies the PC (2') in both states, then the PC binds for $\hat{w}_2 = g(\hat{w}_1)$ in the low-price state $z = 2$, where $g(w)$ is a function implicitly defined by:*

$$u(g(w)) = u(\underline{w}) - (u(w) - u(\underline{w}))\frac{\delta p_1}{1 - \delta p_1}. \qquad (6)$$

*(iii) In the high-price state, $z = 1$, the worker's PC (2') can be satisfied iff $\pi_1 \geq \hat{\pi}_1^{crit} := \underline{w} + c(\alpha h)$. In the low-price state $z = 2$, the PC (2') can be satisfied iff $\pi_1 \geq \hat{\pi}_1^{crit}$ and $\pi_2 \geq \hat{\pi}_2^{crit} := g(\hat{w}_1) + c(\alpha h)$.*

**Proof:** See the appendix.

Lemma 1.1 implies that the worker is willing to work for a wage $g(\hat{w}_1) < \underline{w}$ in the low-price state if $\hat{w}_1 > \underline{w}$. By accepting a wage below $\underline{w}$, the worker can stay in the industry in order to maintain her industry-specific human capital. The wage concession is optimal if the human capital generates rents in the high-price state, i.e. $\hat{w}_1 > \underline{w}$. Given expression (5), it is easy to show that the last inequality requires that some part of the worker's human capital is transferable to other firms in the industry, i.e. $\alpha > 0$.[11] Moreover, the optimal wage concession $\underline{w} - g(\hat{w}_1)$ will increase in the worker's industry-specific human capital $\hat{h} := \alpha h$ since $\partial g(\hat{w}_1)/\partial \hat{h} = -g'(\hat{w}_1)c'(\hat{h}) < 0$, where

$$g'(w) = -\frac{u'(w)}{u'(g(w))}\frac{\delta p_1}{1 - \delta p_1} < 0 \;\; \text{and} \;\; c'(h) = -\frac{c(h)^2}{\bar{c}} < 0.$$

Part (iii) of Lemma 1.1 states values that the industry's output must exceed for employment in the industry to dominate employment outside the industry, such that the

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[10]For ease of exposition, we number lemmas and corollaries according to the Proposition to which they are related. For example, lemma 1.2 is the second lemma building to Proposition 1.

[11]If $\alpha = 0$, then the wage $\hat{w}_1$ will at most equal the wage that the worker can earn outside the industry: $\hat{w}_1 = \min[\pi_1 - \bar{c}, \underline{w}] \leq \underline{w}$. If $\hat{w}_1 = \underline{w}$, then the worker will not make any wage concessions: $g(\hat{w}_1) = g(\underline{w}) = \underline{w}$.

worker's PC (2') is satisfied. They follow from the results in the first two parts of Lemma 1, and the fact that the wage $\hat{w}_z$ increases in the price $\pi_z$ according to expression (5). Both critical prices decrease in the worker's industry-specific human capital $\hat{h} := \alpha h$:

$$\frac{\partial \hat{\pi}_1^{crit}}{\partial \hat{h}} = c'(\hat{h}) < 0, \text{ and } \frac{\partial \hat{\pi}_2^{crit}}{\partial \hat{h}} = c'(\hat{h})(1 - g'(\hat{w}_1)) < 0.$$

The higher the worker's industry-specific human capital, the lower can the price of the industry's output fall before the worker will prefer a job outside the industry to working for firms in the industry at a wage $\hat{w}_z$.

The previous paragraphs characterized the critical prices at which other firms in the industry can make an attractive offer to the worker, i.e., one that dominates employment outside the industry. We next consider the worker's participation constraint (PC) in working for the incumbent, i.e. the firm that can benefit from the worker's firm-specific human capital. In state $z$, the incumbent can employ the worker at a wage $w_z$ which satisfies the following PC:

$$E_z := u(w_z) + \delta E \geq \max[u(\hat{w}_z) + \delta \hat{E}, u(\underline{w}) + \delta \underline{E}] = \max[\hat{E}_z, \underline{E}]. \tag{7}$$

The term on the right-hand side is the value of the worker's most attractive outside option, given by the maximum of the values that are compared in conditions (2) and (2'). The left-hand side is the expected utility that the worker obtains by working for the incumbent in the current period, given by the sum of the utility generated by the wage $w_z$ and the value of future employment opportunities:

$$E := p_1 \max[E_1, \hat{E}_1, \underline{E}] + (1 - p_1) \max[E_2, \hat{E}_2, \underline{E}]. \tag{8}$$

$E \geq \hat{E}$ (defined in expression (4)) since working for the incumbent in the current period preserves the worker's firm-specific human capital beyond this period, which cannot make

the worker worse off.[12]

In order to analyze whether the PC (7) can be satisfied, we first define the highest possible wage for which the incumbent can profit from employing the worker. The incumbent's reservation wage will be denoted as $\bar{w}_z$ and will be determined by the following condition, discussed below:

$$\pi_z - c(h) - \bar{w}_z = -R/(1+f). \tag{9}$$

The term on the left-hand side of condition (9) is the incumbent's profit in a period in which the output price is $\pi_z$ and the incumbent hires the worker to produce output at a total cost of $c(h) + \bar{w}_z$. The term on the right-hand side is the loss that the incumbent incurs if the worker does not produce output for the incumbent and thus looses (at least) her firm-specific human capital.[13] The loss will be strictly positive if the incumbent can extract rents $R > 0$ from the worker's human capital. Condition (9) balances the incumbent's loss of rents and the loss from having the worker produce output. The condition also allows for a cost $f \geq 0$ that the incumbent incurs to finance a loss. By introducing the financing cost, we follow the lead of Froot, Scharfstein and Stein (1993).[14] Net of the financing cost, the incumbent will be willing to take a loss of $R/(1+f)$. By treating the financing cost $f$ as a constant, we assume that this cost has the following structure: given a loss $l$, the financing cost will be

$$\phi(l) := f \max[l, 0], \tag{10}$$

where $f \geq 0$.

We next analyze whether the incumbent's reservation wage $\bar{w}_z$ satisfies the worker's PC (7). The next two results show which of the worker's two outside options (i.e., $\hat{E}_z$ and

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[12]If the worker earned rents due to her firm-specific human capital, then $E > \hat{E}$.

[13]In Section 2, we assumed that the worker must produce output for the incumbent in order to maintain her firm-specific human capital. However, the worker's industry-specific human capital also requires maintenance. We will show that no firm in the industry will hire the worker to produce output if the incumbent does not do so. As a consequence, the worker's industry-specific human capital will also be lost once the worker stops producing output for the incumbent.

[14]The financing cost will yield a rationale for risk management, analyzed in Section 5.

$\underline{E}$) will dominate the other if the constraint binds. We start with the following observation:

**Lemma 1.2:**

*The incumbent's reservation wage exceeds the wage that the worker will be offered by other firms in the industry: $\bar{w}_z \geq \hat{w}_z$ for any $h > 0$. The incumbent can therefore offer the worker more attractive employment than any other firm in the industry:*

$$\bar{E}_z := u(\bar{w}_z) + \delta E \geq u(\hat{w}_z) + \delta \hat{E} = \hat{E}_z, \tag{11}$$

**Proof:** Rearranging condition (9) yields $\bar{w}_z = \pi_z - c(h) + R/(1+f)$. Given the expression (5) for the wage $\hat{w}_z$, we need to distinguish between two cases. If $\pi_z \leq \underline{w} + \bar{c}$, then $\hat{w}_z = \pi_z - c(\alpha h)$ and $\bar{w}_z \geq \hat{w}_z$ since $R/(1+f) \geq 0$ and $c(h) \leq c(\alpha h)$. If $\pi_z > \underline{w} + \bar{c}$, then $\hat{w}_z = \underline{w} + \bar{c} - c(\alpha h)$ and $\bar{w}_z > \underline{w} + \bar{c} - c(h) + R/(1+f) > \hat{w}_z$ since $R/(1+f) \geq 0$ and $c(h) \leq c(\alpha h)$. $\bar{w}_z \geq \hat{w}_z$ implies condition (11) since $E \geq \hat{E}$, as noted below expression (8). $\square$

The above-stated result is intuitive: the incumbent can match any wage offer $\hat{w}_z$ that the worker receives from another firm in the industry because some of the worker's human capital is firm-specific.[15]

We next derive the critical prices of the industry's output for which the incumbent's reservation wage $\bar{w}_z$ just satisfies the worker's PC (7). Given the result that $\bar{E}_z \geq \hat{E}_z$, the PC (7) will take the following form if it binds for the reservation wage $\bar{w}_z$:

$$\bar{E}_z := u(\bar{w}_z) + \delta E = u(\underline{w}) + \delta \underline{E} = \underline{E}, \tag{12}$$

since the right-hand side of condition (7) will then be $\max[\hat{E}_z, \underline{E}] = \underline{E}$. That is, the worker's PC (7) will bind if the reservation wage $\bar{w}_z$ is just high enough to keep the worker

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[15]Recall that, as defined in expression (5), the wage $\hat{w}_z$ may be negative. The result in Lemma 1.2 generalizes to alternative models in which the worker can develop firm-specific human capital after starting to work for a new firm, as long as it takes some time for the human capital to develop. We are essentially assuming that it takes an infinite amount of time, i.e. the worker's firm-specific human capital is lost for good after the worker leaves the incumbent. We will subsequently model human capital formation through training.

from switching to a job outside the industry.

**Lemma 1.3:**

*(i) In the high-price state $z = 1$, the incumbent's reservation wage $\bar{w}_1$ satisfies the worker's PC (7) if $\pi_1 \geq \pi_1^{crit} := \underline{w} + c(h) - R/(1 + f)$.*

*(ii) In the low-price state $z = 2$, the incumbent's reservation wage $\bar{w}_2$ satisfies the worker's PC (7) if $\pi_1 \geq \pi_1^{crit}$ and $\pi_2 \geq \pi_2^{crit} := g(w_1) + c(h) - R/(1 + f)$, where $w_1$ denotes the wage that the worker earns by working for the incumbent in the high-price state, and the function $g(\cdot)$ is defined in Lemma 1.1.*

*(iii) For any $h > 0$, $\pi_z^{crit} \leq \hat{\pi}_z^{crit}$ in both states $z = 1, 2$.*

**Proof:** See the appendix.

The ranking of the critical prices $\pi_z^{crit}$ and $\hat{\pi}_z^{crit}$ in part (iii) of Lemma 1.3 implies that the incumbent can profit from hiring the worker when other firms in the industry cannot do likewise. This result is intuitive because the incumbent is the only firm in the industry which can profit from the worker's firm-specific human capital. As discussed below Lemma 1.2, the incumbent's reservation wage $\bar{w}_z$ will therefore exceed the wage $\hat{w}_z$ that the worker is offered by other firms in the industry.

We next derive the wage that the incumbent will actually offer the worker, and derive conditions for the worker being exposed to wage risk. As discussed above, we assume that the incumbent can make a take-it-or-leave-it offer to the worker. The assumption implies that the offer will just satisfy the worker's PC (7), i.e.

$$E_z := u(w_z) + \delta E = \max[\hat{E}_z, \underline{E}], \tag{13}$$

Since the incumbent always outbids other firms in the industry, all the incumbent must do is match other firms' offers.

Proposition 1 states the result. We impose the condition that $\pi_1 \geq \pi_1^{crit}$ since, otherwise, the incumbent's reservation wage will not satisfy the worker's PC (7) in any state and no one would be able to profitably hire the worker.

**Proposition 1:** *Suppose that $\pi_1 \geq \pi_1^{crit}$. Under-short term contracting, the incumbent will profit from satisfying the worker's PC (7) by paying her the following wages:*

(i) *If $\hat{\pi}_1^{crit} > \pi_1 \geq \pi_1^{crit}$, the worker will work for the incumbent at a wage of $w_1 = \underline{w}$ in the high-price state $z = 1$. If, in addition, $\pi_2 \geq \pi_2^{crit}$, the worker will also work for the incumbent in the low-price state $z = 2$ and will earn a wage of $w_2 = \underline{w}$.*
*The worker will not earn rents, i.e. $E = \underline{E}$.*

(ii) *If $\pi_1 \geq \hat{\pi}_1^{crit}$, the worker will work for the incumbent at a wage of $w_1 = \hat{w}_1$ in the high-price state $z = 1$, where $\hat{w}_z$ is the wage offered by other firms in the industry when $z = 1$. If, in addition, $\pi_2 \geq \pi_2^{crit}$, the worker will also work for the incumbent in the low-price state $z = 2$ and will earn the following wages: $w_2 = g(\hat{w}_1) \leq \underline{w}$ for $\pi_2 < \hat{\pi}_2^{crit}$ and $w_2 = \hat{w}_2$ for $\pi_2 \geq \hat{\pi}_2^{crit}$.*
*If $\pi_1 > \hat{\pi}_1^{crit}$ the worker will earn rents, i.e. $E > \underline{E}$. If $\pi_1 = \hat{\pi}_1^{crit}$, then $E = \underline{E}$.*

**Proof:** See the appendix

A graphical depiction of Proposition 1 is shown in Figure (1). The figure shows $\pi_1$ in the x-axis and $\pi_2$ in the y-axis. $\pi_1^{crit}$ is the minimum price necessary for the incumbent to make a profit in the high-price state. Below this price neither the incumbent nor anyone else in the industry can profitably employ the worker and thus the worker receives no offer. $\hat{\pi}_1^{crit}$ is the minimum price necessary for firms in the industry other than the incumbent to profit from employing the worker in the high-price state. As long as $\pi_1^{crit} \leq \pi \leq \hat{\pi}_1^{crit}$, the incumbent can employ the worker by matching the wage $\underline{w}$ that the worker would receive outside the industry (case (i) of Proposition 1). If $\pi_1 > \hat{\pi}_1^{crit}$, the incumbent has to offer a wage $w_1 = \hat{w}_1$ that matches the offers the worker receives from other firms in the industry (case (ii) of Proposition 1).

The worker's wage in the low-price state $z = 2$ depends on the relation between $\pi_2$, $\pi_2^{crit}$, and $\hat{\pi}_2^{crit}$. If $\pi_2 < \pi_2^{crit}$ then the incumbent's reservation wage $\bar{w}_2$ will not satisfy the worker's participation constraint (7). If $\pi_2^{crit} \leq \pi_2 \leq \hat{\pi}_2^{crit}$ (case (ii) in the Proposition, first part) the incumbent gets to make an aggressive offer to the worker since other firms in

the industry cannot profit from employing the worker at a wage the worker would accept. This changes when $\pi_2 > \hat{\pi}_2^{crit}$ (case (ii) in the Proposition, second part), in which case the incumbent needs to increase the worker's wage in the low-price state to match the offers the worker receives from other firms in the industry.

The results in Proposition 1 can be used to characterize the worker's exposure to risk under short-term contracting. The worker faces two types of risk: on-the-job wage risk, and the risk of a loss of rents associated with human capital when her employment relationship with the incumbent ends. We focus on the first type of risk because our model cannot be used to analyze the second type. To do so would require a model of the correlation between the business conditions in the industry and those outside the industry.

**Corollary 1.1:** *Suppose that $\pi_z \geq \pi_z^{crit}$ in any state $z$. Then, the worker bears on-the-job wage risk ($w_1 > w_2$) iff $\pi_1 \geq \hat{\pi}_1^{crit} = \underline{w} + c(\alpha h)$ and $\pi_2 < \underline{w} + \bar{c}$.*

**Proof:** It is easy to check that, in all cases defined in Proposition 1, $w_1 \geq w_2$. In case (i), $w_1 = \underline{w} = w_2$ such that the worker is not exposed to risk. In case (ii), there is one subcase without wage risk: $\hat{w}_2 = \hat{w}_1$ when $\pi_2 \geq \underline{w} + \bar{c}$ (by expression (5) and $\pi_1 > \pi_2$).[16] The worker's wage income therefore varies across states iff $\pi_1 \geq \hat{\pi}_1^{crit}$ and $\pi_2 < \underline{w} + \bar{c}$. The critical value $\hat{\pi}_1^{crit} = \underline{w} + c(\alpha h)$ was defined in Lemma 1. $\qquad\square$

To discuss the results in Corollary 1.1, we refer to the two cases defined in Proposition 1. In case (i), the worker always earns a wage of $\underline{w}$, either by working for the incumbent, or by working outside the industry. The worker does not bear any risk because the incumbent will never have to compete with other firms in the industry in order to hire the worker. Such competition will, however, occur in case (ii) of Proposition 1. As a consequence, the worker's wage income may vary across states.

We next turn to the effect of the worker's human capital on her exposure to wage risk. Corollary 1.2 below shows that we need to distinguish between three types of industries: high-margin ($\pi_1, \pi_2 \geq \underline{w} + \bar{c}$), low-margin ($\pi_1, \pi_2 < \underline{w} + \bar{c}$), and industries in which margins

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[16]Another possible subcase of case (ii) without wage risk would be $w_1 = \underline{w} = g(\underline{w}) = w_2$, which is, however, inconsistent with our assumption that $\pi_1 > \pi_2$. If we relaxed the assumption in order to allow for $\pi_1 = \pi_2$, then $w_1 = \underline{w} = g(\underline{w}) = w_2$ requires that $\pi_1 = \pi_2 = \hat{\pi}_1^{crit}$. In this case, we would have to slightly modify Corollary 1.1 in order to require that $\pi_1 > \hat{\pi}_1^{crit}$.

are sometimes high and sometimes low $(\pi_1 \geq \underline{w} + \bar{c} > \pi_2)$.

**Corollary 1.2:** *The worker's exposure to on-the-job wage risk will increase in her transferable human capital, i.e. the product $\alpha h$. Specifically,*

*(i) if $\pi_1, \pi_2 \geq \underline{w} + \bar{c}$, then $w_1 = w_2$ (i.e., no on-the-job wage risk),*

*(ii) if $\pi_1 \geq \underline{w} + \bar{c} > \pi_2$, then $w_1 > w_2$ for any $\alpha h > 0$,*

*(iii) if $\pi_1, \pi_2 < \underline{w} + \bar{c}$, then $w_1 > w_2$ iff $\alpha h$ is sufficiently high so that $\pi_1 \geq \hat{\pi}_1^{crit} = \underline{w} + c(\alpha h)$.*

*In cases (ii) and (iii), $w_1 - w_2 = \min[\pi_1, \underline{w} + \bar{c}] - \max[\pi_2, \hat{\pi}_2^{crit}]$, and the worker's exposure to risk will increase in her industry-specific human capital $\hat{h} := \alpha h$:*

$$
\frac{\partial}{\partial \hat{h}} w_1 - w_2 = \begin{cases} 0 & \text{if } \pi_2 \geq \hat{\pi}_2^{crit}, \\ -\frac{\partial \hat{\pi}_2^{crit}}{\partial \hat{h}} > 0 & \text{if } \pi_2 < \hat{\pi}_2^{crit}. \end{cases}
$$

**Proof:** See the appendix.

Corollary 1.2 shows that the worker's human capital will not affect her exposure to wage risk in high-margin industries (case (i)). In all other cases, the worker's exposure to wage risk depends on her *transferable* human capital $\alpha h$, i.e. that part of her human capital which would survive a within-industry job switch. Human capital which is specific to the worker's incumbent employer does not matter because the wage risk is driven by the wage offers that the worker receives from other firms in the industry, and those wage offers depend on the transferable part of the worker's human capital, i.e. the part from which the other firms would benefit.

The extent of the worker's exposure to wage risk also depends on industry characteristics. In low-margin industries (case (iii)), a worker will only be exposed to wage risk if she has sufficient transferable human capital $\alpha h$. The extent of the exposure will increase in the product $\alpha h$ until the worker bears the entire risk of changes in the industry's output price: $w_1 - w_2 = \pi_1 - \pi_2$. In industries in which margins are sometimes high and sometimes low (case (ii)), the worker will always be exposed to wage risk, but the extent of the

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exposure will again increase in the worker's transferable human capital $\alpha h$ until it reaches a maximum above the maximum exposure that is possible in low-margin industries.[17]

The characterization of the worker's exposure to on-the-job wage risk in Corollary 1.2 reveals that the financing cost $f$ does not affect the exposure under short-term contracting since it does not affect any of the critical prices that appear in Corollary 1.2, i.e. the critical prices $\hat{\pi}_1^{crit}$ and $\hat{\pi}_2^{crit}$.[18] The reason is that the financing cost does not affect the wage offers that the worker receives from firms in the industry other than the incumbent, i.e. the wage $\hat{w}_z$ defined in expression (5).

Corollary 1.2 characterizes the effect of the worker's human capital on her exposure to wage risk for exogenous values of the price of the industry's output, $\pi_1$ and $\pi_2$. Section 4 will present an extension of our model in which the prices $\pi_1$ and $\pi_2$ are jointly determined with the worker's human capital in an industry equilibrium. Given the topic of the paper, we will focus on parameterizations of the (extended) model for which the worker is exposed to wage risk. We will do so by imposing parameter restrictions such that the prices $\pi_1$ and $\pi_2$ satisfy the conditions stated in Lemma 1.1, which are necessary and sufficient for the worker being exposed to wage risk:

**Assumption 1:** *Under short-term contracting, the worker is exposed to on-the-job wage risk:*

$$\pi_1 \geq \hat{\pi}_1^{crit} = \underline{w} + c(\alpha h) \text{ and } \underline{w} + \bar{c} > \pi_2 \geq \pi_2^{crit} = g(\hat{w}_1) + c(h) - R/(1+f). \quad (14)$$

In stating Assumption 1, we have restated the definitions of $\hat{\pi}_1^{crit}$ (in part (iii) of Lemma 1.1) and $\pi_2^{crit}$ (in Lemma 1.3) and have used that $g(w_1) = g(\hat{w}_1)$ since the first condition implies $w_1 = \hat{w}_1$ (by Proposition 1). The requirement $\pi_1 \geq \hat{\pi}_1^{crit}$ focuses the subsequent analysis on cases in which the incumbent will compete with other firms in the

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[17]In case (ii), the maximum of $w_1 - w_2$ is $\underline{w} + \bar{c} - \pi_2$ which exceeds the price difference $\pi_1 - \pi_2$ since case (ii) is defined by $\pi_1 \geq \underline{w} + \bar{c}$.

[18]By Lemma 3, the financing cost $f$ does increase the critical prices $\pi_1^{crit}$ and $\pi_2^{crit}$ below which the incumbent will stop employing the worker, but the worker's exposure to *on-the-job* wage risk will not depend on $f$.

industry in order to employ the worker in the high-price state. In such cases, the other firms would (obviously) profit from employing the worker,[19] but the incumbent will match their wage offers in order to earn a higher profit. The requirement that $pi_1 \geq \hat{\pi}_1^{crit}$ therefore rules out that the incumbent may have to finance a loss in the high-price state $z = 1$. In the low-price state $z = 2$, the incumbent may incur a loss in order to avoid a loss of future rents, but such a loss will only occur in cases in which no other firm in the industry can hire the worker at the wage $\hat{w}_2$. If another firm could profit from employing the worker at the wage $\hat{w}_2$, then the incumbent can also profit from matching this wage.[20] We can therefore conclude that the incumbent will only incur a loss in situations in which the worker is willing to make a wage concession by accepting a wage $w_2 = g(w_1) < \underline{w}$.

## 4  Short-term contracting in industry equilibrium

We now extend the model in order to analyze the structure of the industry in terms of worker human capital. To do so, we add the following assumptions:

**Assumption 2:** *Firms will enter into the industry until the expected profit from entry is zero. All firms take the prices of the industry's output as given. The prices are determined by the following inverse demand function:*

$$\pi_z = a_z - b\,Q_z, \tag{15}$$

*where $Q_z$ denotes the aggregate output of the industry in state $z$, and $a_1 > a_2$. The average price of the industry's output is $\bar{\pi} = p_1\pi_1 + p_2\pi_2$.*

**Assumption 3:** *Upon entry, firms can train workers. Training generates human capital $h$ at a training cost $T(h) := \frac{1}{2}\tau h^2$, where $\tau$ is a constant that regulates the sensitivity of*

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[19]Recall that the critical price $\hat{\pi}_1^{crit}$ is defined in Lemma 1.1 as the price for which the wage $\hat{w}_1$ just satisfies the worker's PC (2) in the high-price state $z = 1$. Moreover, recall that, as discussed above equation (5, the wage $\hat{w}_1$ is defined so that firms in the industry other than the incumbent will never make a loss by offering this wage if $z = 1$. In summary, the firms would therefore profit from employing the worker at the wage $\hat{w}_1$, if the worker were to leave the incumbent.

[20]To see this, first note that, by expression (5), $\pi_2 - \hat{w}_2 - c(\alpha) \geq 0$. The latter inequality implies that the incumbent's profit from matching the wage $\hat{w}_2$ is $\pi_2 - \hat{w}_2 - c(h) \geq 0$ since $c(h) \leq c(\alpha h)$.

*training costs to the level of human capital. If $T(h)$ is spent on training a worker in some operating period, then the worker's human capital increases by $h$ at the start of the next operating period. The human capital consists of firm-specific and industry-specific human capital, as described below expression ([1](#)) in Section 2. The firm-specific human capital is associated with the firm that trained the worker. Other firms can observe the extent to which the worker was trained.*

Given that our model features constant returns to scale, it does not specify the number of workers per firm. We can, however, derive the number of workers who receive training, which will be denoted as $n_t$. Given Assumption 1, the trained workers will produce output in every operating period. In addition, output may be produced by untrained workers without human capital: let $n_z$ denote the number of untrained workers who produce output in state $z \in \{1, 2\}$.

We now formally define the industry equilibrium. The definition will be discussed below.

**Industry equilibrium:** *Given Assumption 1, an industry equilibrium is characterized by a number $n_t$ of trained workers, as well as the number $n_z$ of untrained workers who produce output in state $z \in \{1, 2\}$. The trained workers are trained to an optimal extent $h^*$ given by the following objective function:*

$$h^* = \arg \max_h R(h) - T(h), \tag{16}$$

*where*

$$R(h) := \frac{\delta}{1 - \delta} \left( \bar{\pi} - \bar{w} - c(h) - p_2 \phi(w_2 + c(h) - \pi_2) \right), \tag{17}$$

*$\bar{\pi}$ is the average price of the industry's output across the two states $z = 1, 2$, $c(h)$ is given by ([1](#)), and $\bar{w}$ is the average wage that a trained worker receives: $\bar{w} = p_1 w_1 + p_2 w_2$, given the wages $w_1$ and $w_2$ specified in Proposition 1. The average price $\bar{\pi}$ is defined in Assumption 2 as the average of the prices of the industry's output in the states $z = 1, 2$,*

*where $Q_z = n_t + n_z$, and $n_t$ and $n_z$ satisfy the zero-profit conditions:*

$$n_z = \min[(a - \underline{w} - \bar{c})/b - n_t, 0], \tag{18}$$

$$R(h^*) = T(h^*). \tag{19}$$

Condition (16) requires that any trained worker is trained to an optimal extent. The optimal training level maximizes the difference $R(h) - T(h)$ between the present value of the rents that a firm can earn by training a worker and a training cost $T(h)$. The present value of the rents is defined in expression (4). In any operating period, the expected rent consists of the average profit from selling the worker's output, $\bar{\pi} - \bar{w} - c(h)$, minus the expected cost of financing losses in the low-price state $z = 2$.[21]

Conditions (4) and (18) are zero-profit conditions that determine the extent to which workers enter the industry. The Condition (4) requires that the present value of the expected rents generated by an optimally trained worker's human capital equal the cost of the worker's training. Condition (18) requires that, in state $z$, untrained workers will enter the industry in order to produce output at a total cost of $\underline{w} + \bar{c}$ if the trained workers' aggregate output is too small to drive the output price down to the cost $\underline{w} + \bar{c}$. In this case, the untrained workers will produce sufficient output so that

$$\pi_z = a - b(n_t + n_z) = \underline{w} + \bar{c}.$$

Solving the last equation for $n_z$ yields the first term stated inside the minimum function in condition (18). If $\pi_z = a - bn_t < \underline{w} + \bar{c}$, then no untrained workers will enter the industry:

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[21]In defining the function $R(h)$, we assume that a trained worker will produce output in every future period, which is an implication of Assumption 1. Recall that Assumption 1 focuses our analysis on a trained worker's exposure to on-the-job wage risk. By definition, on-the-job wage risk is wage risk that the worker bears while on the job. Assumption 1 therefore puts the focus on cases in which a trained worker will always stay employed by the firm that trained the worker. In such cases, the firm will benefit in every future period from the effect of training on the cost $c(h)$ at which the worker can produce output. As discussed at the end of Section 3, Assumption 1 also implies that no losses will have to be financed in the high-price state $z = 1$.

$n_z = 0$.

The next steps consist of finding the values for $h$ and $\pi_z$ that would take place in equilibrium, and a corresponding analysis of the relation between human capital and the risk workers must bear. These steps are work in progress which will be added to a later version of the paper.

# 5   Long-term contracting

A long-term contract is a wage schedule $(W_1, W_2)$ which specifies wages that the worker receives if the worker ends up working for the incumbent in a specific state. We will analyze long-term contracting in an incomplete contracts setting which is specified as follows:

**Assumption 2:** *Long-term contracting requires a skill ("trust") that is part of the worker's firm-specific human capital. Long-term contracting is about wages, but not about employment: neither can a firm commit to actually employing the worker, nor can the worker commit to actually working for a firm. However, the parties to a long-term contract are bound by the contract if they continue their employment relationship. If a party to a long-term contract wants to terminate the contract, then this party must end the underlying employment relationship.*

Assumption 2 deviates from a literature following Holmstrom (1983) in which it is assumed that firms can fully commit to employing workers under a pre-determined wage policy, but workers cannot commit to abstain from quitting. We follow Berk et al. (2010) in that we relax the assumption of full commitment on part of the firms. We thus obtain a setting in which both parties to a long-term contract have symmetric commitment limitations.

Assumption 2 also specifies how long-term contracting is related to human capital. We assume that such contracting is only possible if the worker involved is willing to trust her employer. Trust is required because the employer can only benefit from offering a long-term contract if the contract reduces the worker's wage in one state. For the worker to

accept the contract, the wage reduction must be tied to the promise of a wage increase that the worker must be able to trust. We assume that trust is part of the worker's firm-specific human capital. The assumption implies that trust is perishable in the same way as human capital: it will only continue to exist as long as the worker and the incumbent continue their employment relationship. Once the worker has lost her firm-specific human capital, she will only accept the wages offered to her in short-term contracting, i.e. the wages we derived in Section 3.[22]

As in the last section, we assume that wages are set by the incumbent through a take-it-or-leave-it offer to the worker. The following program, discussed below, characterizes the incumbent's offer:

$$\min_{W_1,W_2} \ Z(W_1,W_2) := p_1 W_1 + p_2 W_2 + F^L(W_1,W_2) \quad (20)$$

subject to

Worker's PC: $\qquad\qquad\qquad E^L \geq E, \qquad\qquad\qquad (21)$

Worker's ICs, $ICW_z$: $\quad E_z^L := u(W_z) + \delta E^L \geq \max[\hat{E}_z, \underline{E}] = E_z, \qquad (22)$

Incumbent's ICs, $ICI_z$: $\qquad W_z \leq \bar{W}_z := \pi_z - c(h) + R^L/(1+f), \qquad (23)$

where the last two lines specify constraints that must hold in both states $z = 1, 2$.

The objective function $Z(W_1, W_2)$ is the sum of the incumbent's expected wage payments and expected costs of financing any losses that the incumbent may incur due to the employment relationship under long-term contracting. For now, the expected financing cost is simply denoted as $F^L(W_1, W_2)$.

The incumbent's take-it-or-leave-it offer will only be acceptable to the worker if she is no worse off than under short-term contracting, which is formally stated in condition (21). We assume that the worker must accept the offer before knowing the state. In condition (21), $E^L$ denotes the expected utility that the worker obtains by accepting the incumbent's offer before the state is known. The condition is a participation constraint

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[22]The assumption thus deals with the issue of multiple equilibria that arises in infinitely repeated games.

(PC) which requires that $E^L$ exceeds the expected utility under short-term contracting, $E$, defined in expression (8).

In stating the objective function, we assume that, in offering the long-term contract, the incumbent specifies wages for both states. If, instead, the long-term contract only specifies a wage for one state $z = i$, then the worker and the incumbent would have to negotiate a short-term contract in the other state. It is easy to show that, in this case, long-term contracting would be equivalent to short-term contracting, because the wage set through long-term contracting for state $i$ would have to be set such that condition (21) binds. A similar argument rules out long-term contracts under which it is not incentive compatible for one party to continue the employment relationship once the state is known. Therefore, the above-stated program contains four incentive compatibility (IC) conditions, discussed next.

Condition (22) requires that, in state $z = 1, 2$, the worker must be willing to work for the incumbent at the wage $W_z$, rather than either switching to another firm in the industry or taking a job outside the industry. The left-hand side of the condition is the expected utility that the worker obtains from employment with the incumbent in the current period, where

$$E^L := p_1 \max[E_1^L, \hat{E}_1, \underline{E}] + (1 - p_1) \max[E_2^L, \hat{E}_2, \underline{E}], \tag{24}$$

The right-hand side of condition (22) represents the outside option that the worker obtains upon terminating the long-term contract. By Assumption 2, terminating the long-term contract requires termination of the employment relationship with the incumbent. Given our assumptions regarding the perishability of the worker's human capital (including "trust"), the worker's outside options are given by the analysis of short-term contracting, i.e. the values $\hat{E}_z$ and $\underline{E}$ defined in expressions (2') and (3), respectively. The last equation in condition (22) follows from condition (7), i.e. the condition which requires that, under short-term contracting, the incumbent offers employment more attractive than the worker's outside option. Condition (22) therefore requires that employment under long-term contracting is

*in each state* more attractive than employment under short-term contracting.

Condition (23) requires that the incumbent must be willing to actually employ the worker at the wage $W_z$, which requires that this wage is below the incumbent's reservation wage in state $z = 1, 2$. The definition of the reservation wage $\bar{W}_z$ in condition (23) differs from that of the reservation wage $\bar{w}_z$ (in expression (9)) in that we allow for a different value of rents that the incumbent earns under long-term contracting relative to short-term contracting. We denote the value of the rents earned under long-term contracting as $R^L$. The definition of the reservation wage $\bar{W}_z$ in expression (23) follows from a condition similar to condition (9).

We next characterize optimal long-term contracts. A natural starting point is a long-term contract which specifies the wages that result from short-term contracting. Lemma 4 therefore compares the incumbent's expected wage bill under a long-term contract to that under short-term contracting.

**Lemma 3.1:** *Long-term contracting will reduce the incumbent's expected wage bill if $W_1 \leq w_1$ and $W_2 \geq w_2$ and $W_1 \geq W_2$. If $W_1 < w_1$, then $W_2 > w_2$ and long-term contracting will strictly reduce the expected wage bill.*

**Proof:** See the appendix.

Lemma 3.1 implies that the incumbent faces a trade-off between reducing the expected wage bill and reducing expected loss financing costs $F^L(W_1, W_2)$. Reducing the expected wage bill requires a long-term contract which specifies wages $W_1 < w_1$ and $W_2 > w_2$. As a consequence, the incumbent's profit in the high-price state will be higher than under short-term contracting, but that in the low-price state will be smaller. The latter effect matters since, as discussed at the end of Section 3, the incumbent may have to finance losses from employing the worker in the low-price state. With $W_2 \geq w_2$, a long-term contract increases the incumbent's wage bill in state $z = 2$, and may thus increase the incumbent's expected loss financing costs.

We next examine which of the constraints (21)-(22) will always bind at the optimum

of the long-term contracting problem (20)-(22), and which constraints will never bind. We use subscripts to refer to the constraints that must be satisfied in some state $z$ for the incumbent and the worker to continue their employment relationship in this state. For example, $ICW_1$ is the worker's IC in the high-price state, given by equation (22) with $z = 1$. More generally, $ICI_z$ and $ICW_z$ are defined in expressions (23) and (22), respectively.

**Lemma 3.2:** *(i) If $W_1 = w_1$ and $W_2 = w_2$, then the PC (21) and the constraints $ICW_1$ and $ICW_2$ bind.*

*(ii) If $W_1 < w_1$ and $W_2 > w_2$, then the PC (21) and the constraints $ICI_1$ and $ICW_2$ do not bind, but the constraint $ICW_1$ binds.*

*(iii) If $W_1 > w_1$ and $W_2 < w_2$, then the PC (21) and the constraints $ICI_1$ and $ICW_1$ do not bind, but the constraint $ICW_2$ binds.*

**Proof:** See the appendix.

Lemma 3.2 lists three cases. In the first case, the long-term contract specifies the same wages as those obtained under short-term contracting. This case is the only case in which the long-term contract satisfies the worker's PC (21) (and all other constraints) while setting $W_z \leq w_z$ in both states $z = 1, 2$. Given that it is never optimal for the incumbent to offer a long-term contract featuring $W_z \geq w_z$ in both states, the two other cases in Lemma 3.2 cover all remaining possible contracting outcomes. In case (ii), the contract transfers risk from the worker to the incumbent by reducing the worker's wage in the high-price/high-wage state $z = 1$ and increasing the wage in the low-price/low-wage state $z = 2$. In case (iii), the contract may save costs of financing a loss that the incumbent would incur under short-term contracting in the low-price state.

Lemma 3.2 shows that the optimal long-term contract will always be constrained by the constraint (22) which represents the need to keep the worker from ending her employment relationship with the incumbent in state $z$. The constraint will bind in different states, depending on the way in which the long-term contract deviates from short-term contracting. As a consequence, the optimal long-term contract may specify the same

wages as those obtained under short-term contracting.

To see this, consider changing the wages $W_1$ and $W_2$ relative to the wages $w_1$ and $w_2$ defined in Proposition 1. The latter wages satisfy the constraint (22) in both states since they are determined by equation (13). Suppose that the wages are changed infinitesimally by $dW_1 = W_1 - w_1$ and $dW_2 = W_2 - w_2$, where setting $dW_1 < 0$ and $dW_2 > 0$ corresponds to case (ii) of Lemma 3.2, and setting $dW_1 > 0$ and $dW_2 < 0$ corresponds to case (iii). Given the results in Lemma 3.2, the wage changes must satisfy the following condition for the constraint (22) to continue to hold in both states:[23]

$$dW_1 = -\frac{u'(w_2)}{u'(w_1)}\frac{\delta p_2}{1-\delta+\delta p_1}dW_2 \quad \text{if } dW_1 < 0 \text{ and } dW_2 > 0,$$
$$dW_1 = -\frac{u'(w_2)}{u'(w_1)}\frac{1-\delta+\delta p_2}{\delta p_1}dW_2 \quad \text{if } dW_1 > 0 \text{ and } dW_2 < 0,$$

Given the incumbent's objective function (20), the wage changes are benefitial if the following condition holds:

$$dZ = p_1 dW_1 + p_2 dW_2 + dF^L < 0,$$

where $dF^L$ denotes the change in the incumbent's expected cost of financing any losses incurred under the employment contract. Stated in terms of the financing cost function (10), $dF^L = p_2\phi'(w_2 + c(h) - \pi_2)dW_2$ since the incumbent may have to finance losses only in the low-price state which occurs with probability $p_2$.

Upon substituting the above-stated relations between the wage changes $dW_1$ and $dW_2$ into the expression for $dZ$, we find that the wage changes benefit the incumbent (i.e., $dZ < 0$) if the following conditions hold:

for $dW_1 < 0$ and $dW_2 > 0$, $dZ > 0$ if $\quad (1 + \phi'(w_2 + c(h) - \pi_2))\frac{1-\delta+\delta p_1}{\delta p_1} < \frac{u'(w_2)}{u'(w_1)}.$ (25)

for $dW_1 < 0$ and $dW_2 > 0$, $dZ > 0$ if $\quad (1 + \phi'(w_2 + c(h) - \pi_2))\frac{\delta p_2}{1-\delta+\delta p_2} > \frac{u'(w_2)}{u'(w_1)}$ (26)

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[23]To obtain the first result, we start with constraint $ICW_1$ (i.e., the $z = 1$ version of constraint (22)) which binds in case (ii) of Lemma 5. The constraint restricts the wage changes by requiring that that $dE_1^L = 0$, where $dE_1^L = u'(w_1)dW_1 + \frac{\delta}{1-\delta}(p_1 u'(w_1)dW_1 + (1-p_1)u'(w_2)dW_2)$ since $E^L = (p_1 u(W_1) + (1-p_2)u(E_2))/(1-\delta)$. Rearranging the condition $dE_L^1 = 0$ yields the first result. The second result is obtained in a similar way, i.e. by rearranging the condition $dE_2^L = 0$.

Condition (25) is obtained by rearranging the condition $dZ < 0$ for the case in which long-term contracting results in wage changes $dW_1 < 0$ and $dW_2 > 0$. If the inequality holds, then the wage change benefits the incumbent by reducing the expected wage bill. In this case, the reduction in the expected wage bill must exceed any increase in the expected cost of financing losses that the incumbent incurs due to the employment relationship. The expected financing cost may increase because the worker will earn a higher wage in the low-price state: $W_2 = w_2 + dW_2 > w_2$. The second inequality in condition (25) therefore requires that the worker's risk-aversion is sufficiently high and/or her exposure to on-the-job wage risk is sufficiently high under short-term contracting.

Condition (26) results from rearranging the condition $dZ < 0$ for the case in which long-term contracting results in wage changes $dW_1 > 0$ and $dW_2 < 0$. If the inequality holds, then the wage changes benefit the incumbent by reducing the expected cost of financing losses that the incumbent incurs in the low-price state. In this case, the expected financing cost reduction must exceed the increase in the expected wage bill that the worker must receive in order to accept a higher exposure to wage risk than under short-term contracting: $W_1 - W_2 > w_1 - w_2$. The first inequality in condition (25) therefore requires that the worker's risk aversion is sufficiently small and/or her exposure to on-the-job wage risk is sufficiently high under short-term contracting.

Given the topic of the present paper, we will focus on the case in which long-term contracting is used to reduce the worker's exposure to wage risk:

**Assumption 4:** *The financing cost function $\phi(l)$ satisfies the following condition:*

$$f < \frac{1 - \delta}{\delta p_2}.$$

Assumption 3 implies that the bound on the marginal rate of substitution $u'(w_2)/u'(w_1)$ in condition (26) will be smaller than one since $\phi'(\cdot) = \max[f, 0]$. As a consequence, condition (26) will be violated for any wages $w_1 \geq w_2$.[24]

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[24]Alternatively, we could assume that the worker is sufficiently risk-averse.

We will next characterize the optimal long-term contract in two steps. We first consider the program (20)-(22) for the case in which the constraint (23) does not bind. In a second step, which is work in progress and will be added in a future version of the paper, we will consider the optimal contract if the constraint (23) binds.

We focus on the case in which condition (25) holds; otherwise, the optimal long-term contract specifies wages equal to those that the worker would earn under short-term contracting, and the worker's exposure to wage risk continues to be characterized by Corollary 1.2.

Proposition 2 states the optimal long-term contract if the contract is constrained by the need to keep the worker from terminating the employment relationship (i.e., the constraint $ICW_1$, which binds according to part (ii) of Lemma 3.2).

**Proposition 2:** *If the optimal long-term contract is constrained by worker incentive compatibility (condition (25)), then $W_1^*$ and $W_2^*$ are given by:*

$$u(W_1^*) + \frac{\delta}{1-\delta}(p_1 u(W_1^*) + p_2 u(W_2^*)) = E_1, \tag{27}$$

*and*

$$\frac{u'(W_2^*)}{u'(W_1^*)} = (1 + \phi'(W_2^* + c(h) - \pi_2))\left(1 + \frac{1-\delta}{\delta p_1}\right). \tag{28}$$

**Proof:** See the appendix.

We show in the proof of Proposition 2 that assuming convexity of the financing cost function $\phi(\cdot)$ is sufficient (but not required) for the contract $(W_1^*, W_2^*)$ to yield a minimum of the incumbent's objective function (20).[25] The value $E_1$ on the right-hand side of the first condition in Proposition 2 is the utility that the worker derives from short-term contracting in the high-price state, which equals the right-hand side of the worker's incentive compatibility constraint $ICW_1$ (i.e., expression (22) with $z = 1$) by condition (12). In the high-price state, the long-term contract must therefore promise the worker a wage

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[25]Our specification of the cost function in (10) satisfies this requirement. The assumption implies that the expected financing cost $F^L(W_1, W_2) = F(W_2)$ will be convex in $W_2$.

$W_1^*$ for which the worker is indifferent between long-term and short-term contracting. The wage $W_1^*$ can be smaller than the wage $w_1$ obtained under short-term contracting because the worker values the reduction in her exposure to risk achieved by long-term contracting.

Figure (2) depicts graphically the result in Proposition 2. Starting from the short-term contract $(w_1, w_2)$, the assumption that the incumbent gets to make take-it-or-leave-it offers implies that the worker will not see her utility from future wages increase relative to her previous expected utility starting from state 1, $E_1$. Thus the optimal long-term contract $W_1^*, W_2^*$ will lie on the indifference curve which represents combinations of $W_z$ such that the worker's expected utility remains $E_1$.[26] The optimum is the point of tangency between the worker's indifference curve and the iso-lines representing the incumbent's objective function. Figure (2) shows the case in which the financing cost is zero.

Condition (28) in Proposition 2 shows that, in contrast to short-term contracting, the worker's exposure to wage risk under long-term contracting will not depend on her human capital if it is costless to finance losses that the incumbent incurs by raising the worker's wage in the low-price/wage state $z = 2$. To see this, notice that with $\phi'(\cdot) = 0$, the right-hand side of condition (28) does neither depend on the worker's human capital $h$, nor on the extent to which the human capital is transferable, $\alpha$.

The irrelevance results will, however, vanish in the presence of financing costs. If $f > 0$, then the transfer of wage risk may be constrained by the cost of financing losses that the incumbent incurs in the state $z = 2$ in which long-term contracting raises the worker's wage. Given a loss of $W_2^* + c(h) - \pi_2 > 0$, condition (28) specifies that the worker's marginal rate of substitution between wage income in the two states is raised by a factor of $(1 + f)$ if $\phi'(W_2^* + c(h) - \pi_2) = f > 0$.

We conclude here this version's analysis of long-term contracts. The analysis of the impact of human capital on the riskiness of human capital will be added on a future version of the paper. The analysis of the industry equilibrium narrows the possible outcomes for prices and wages, and therefore produce more accurate implications for the risk of human

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[26]Her overall utility $E$, and the utility starting from state 2 $E_2$ does increase.

capital than the analysis we had in a previous version of the paper.

# 6    Conclusion

We build a simple model that allows us to characterize the amount of risk-sharing, and the resulting wage-risk a worker will bear, as a function of human capital and industry characteristics. A key characteristic driving risk-sharing is the transferability of human capital, since it determines the magnitudes of the rents firms and workers will earn from having more skills. A second key characteristic, is the variability in the profitability of the industry. These characteristics determine wage-risk and the extent to which firms can offer "Insurance within the firm".

Under short-term contracts, workers will face wage-risk due to their human capital (Proposition 1), and increases in human capital will lead to increases wage-risk (Corollary 1.2). Long-term contracting can raise a worker's expected utility while reducing a firm's wage bill. We show that when firms can commit to a long-term contract, the optimal wage-risk exposure that the worker ends up with does not depend on the level of human capital or its transferability as long as there are no financing costs.

We do not analyze these industry equilibrium effects in this version of the paper, but leave that for a future version. The analysis will yield multiple empirical implications relating wage-risk, risk-sharing within the firm, human capital, and financing costs. Testing the implications from our model in an empirical setting is another natural follow up step from this work.

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# 7 Appendix

## 7.1 Proof of Lemma 1.1

(i) Stated in the form (2'), the PC (2) will bind for $\hat{E}_1 = \underline{E}$. Moreover, it must be that $\hat{E}_2 \leq \underline{E}$ since $\hat{w}_2 \leq \hat{w}_1$ (by expression (5) and $\pi_1 \geq \pi_2$ due to state $z = 1$ being the high-price state). With $\hat{E}_1 = \underline{E}$ and $\hat{E}_2 \leq \underline{E}$, definition (4) implies $\hat{E} = \underline{E}$ and the PC (2) will bind for $\hat{w}_1 = \underline{w}$.

(ii) The PC (2') will never be satisfied only in the low-price state $z = 2$. Suppose the contrary were true, i.e. $\hat{E}_2 \geq \underline{E}$ and $\hat{E}_1 < \underline{E}$. In this case, it must be that $\hat{w}_2 > \hat{w}_1$ since $\hat{E}_z = u(\hat{w}_z) + \delta\hat{E}$. But, $\hat{w}_2 > \hat{w}_1$ is incompatible with definition (5) and $\pi_1 \geq \pi_2$ (i.e., state $z = 1$ being the high-price state).

Next, suppose that the PC (2) is satisfied in both states and it binds for $z = 2$, i.e. $\hat{E}_2 = \underline{E}$. Then, $\hat{E} = p_1\hat{E}_1 + (1 - p_1)\underline{E}$. Substituting the latter expression and expression (3) into the condition $\hat{E}_2 = u(\hat{w}_2) + \delta\hat{E} = \underline{E}$ (i.e. the PC (2') which binds for $z = 2$) shows that the condition holds for $\hat{w}_2 = g(\hat{w}_1)$.

(iii) The critical price $\hat{\pi}_1^{crit}$ solves the condition $\hat{w}_1 = \underline{w}$ stated in part (i) upon using expression (5) to substitute for $\hat{w}_1$. Since $\hat{w}_1$ increases in the price $\pi_1$ by expression (5), $\hat{w}_1 \geq \underline{w}$ iff $\pi_1 \geq \hat{\pi}_1^{crit}$, in which case the PC (2') will hold since $u'(\cdot) > 0$. By part (ii), the condition $\pi_1 \geq \hat{\pi}_1^{crit}$ is necessary for the PC (2') to hold for $z = 2$. The critical price $\hat{\pi}_2^{crit}$ solves the condition $\hat{w}_2 = g(\hat{w}_1)$ stated in part (i) upon using expression (5) to substitute for $\hat{w}_2$. If $\pi_1 \geq \hat{\pi}_1^{crit}$, the PC (2') will hold for $pi_2 \geq \hat{\pi}_2^{crit}$ since $\hat{w}_2$ increases in the price $\pi_2$ by expression (5). $\qquad\square$

## 7.2 Proof of Lemma 1.3

(i) The critical price $\pi_1^{crit}$ is the price of the industry's output in case condition (12) holds in state $z = 1$, i.e. $\bar{E}_1 = \underline{E}$. In this case, $\bar{E}_2 < \max[\hat{E}_2, \underline{E}]$, i.e. the PC (7) must be violated in the low-price state for $w_2 = \bar{w}_2$ (since $\bar{w}_2 < \bar{w}_1$ because $\pi_2 < \pi_1$ and the output price $\pi_z$ increases the reservation wage $\bar{w}_z$ by expression (9)). Conditional on being employed by the incumbent in the current period, the value of the worker's future employment opportunities (defined in expression (8)) will then be given by:

$$E = p_1\bar{E}_1 + (1 - p_1)\underline{E},$$

where we have used that $\max[E_2, \hat{E}_2, \underline{E}] = \underline{E}$ by the following implication of the ranking in expression (11):

$$\bar{E}_z \leq \underline{E} \Rightarrow \hat{E}_z < \underline{E} \Rightarrow \max[E_z, \hat{E}_z, \underline{E}] = \underline{E}. \tag{29}$$

Substituting for $E$ and $\bar{E}$ (defined in expression (3)) in condition (12) shows that the condition reduces to $\bar{w}_1 = \underline{w}$. Using expression (9) to substitute for $\bar{w}_1$ yields $\pi_1^{crit} := \underline{w} + c(h) - R/(1 + f)$.

(ii) Suppose that condition (12) binds in state $z = 2$: $\bar{E}_2 = \underline{E}$. As stated in expression (29), we then obtain that $\max[E_2, \hat{E}_2, \underline{E}] = \underline{E}$ in expression (8). Moreover, since $\bar{w}_2 < \bar{w}_1$, the fact that condition (12) binds in state $z = 2$ implies that there must exist a wage $w_1$ for which PC (7) can be satisfied in state $z = 1$, i.e. $E_1 \geq \max[\hat{E}_1, \underline{E}]$ and, thus, $\max[E_1, \hat{E}_1, \underline{E}] = E_1$.

Using the last insights to substituting into expression (8)) yields

$$E = p_1 E_1 + (1 - p_1)\underline{E}.$$

Upon substituting for $E$ and $\bar{E}$ (defined in expression (3)) in the constraint (12) for $z = 2$, the condition can be rewritten as follows:

$$u(\bar{w}_2) = u(\underline{w}) - (u(w_1) - u(\underline{w}))\frac{\delta p_1}{1 - \delta p_1} = u(g(w_1)), \tag{30}$$

where $g(\cdot)$ is the function defined in Lemma 1. Using expression (9) to substitute for $\bar{w}_2$ yields $\pi_2^{crit} = g(w_1) + c(h) - R/(1+f)$.

(iii) The critical prices $\hat{\pi}_z^{crit}$ are values of the price of the industry's output for which $\hat{E}_z = \underline{E}$ such that the PC (2') binds. Given that $\bar{E}_z \geq \hat{E}_z$ (expression (11)), the incumbent's reservation wage $\bar{w}_z$ will always satisfy the worker's PC (7) if $\pi_z = \hat{\pi}_z^{crit}$. Since the reservation wage $\bar{w}_z$ increases in the price $\pi_z$, the PC (7) must bind for a price $\pi_z^{crit} \leq \hat{\pi}_z^{crit}$. □

## 7.3  Proof of Proposition 1

(i) $\hat{\pi}_1^{crit} > \pi_1^{crit}$ by part (iii) of Lemma 3. For $\hat{\pi}_1^{crit} > \pi_1$, the worker's PC (2') is not satisfied in any state $z$, i.e. $\hat{E}_z < \underline{E}$ (by Lemma 1), and we obtain the following specification of condition (13):

$$E_z = u(w_z) + \delta E = \max[\hat{E}_z, \underline{E}] = \underline{E}.$$

To derive the wage $w_z$, we distinguish between two subcases: (1) $\pi_2 < \pi_2^{crit}$, or (2) $\pi_2 \geq \pi_2^{crit}$. By the definition of the critical price $\pi_2^{crit}$, in subcase (1) the incumbent cannot profit from hiring the worker in the low-price state at any wage that the worker would accept: $\bar{E}_2 < \max[\hat{E}_2, \underline{E}]$. In the current period, working for the incumbent will yield the following value of the worker's future employment opportunities: $E = p_1 E_1 + (1 - p_1)\underline{E}$, where result (29) has been used to obtain that $\max[E_2, \hat{E}_2, \underline{E}] = \underline{E}$. Upon substituting for $E$ and $\underline{E}$ (defined in expression (3)) in the above-stated specification of condition (13), the condition requires $u(w_1) = u(\underline{w})$, i.e. $w_1 = \underline{w}$.

In subcase (2), the incumbent can profit from hiring the worker both states (by Lemma 3), and the above-stated specification of condition (13) implies that the incumbent will always offer the worker a wage $w_z$ for which $E_z = \underline{E}$. Thus, we again obtain that $E = \underline{E}$ and, in both states $z = 1, 2$, $E_z = u(w_z) + \delta \underline{E} = \underline{E}$, which implies that $w_1 = w_2 = \underline{w}$ by the definition of $\underline{E}$ in expression (3).

(ii) For $\pi_1 \geq \hat{\pi}_1^{crit}$, $\hat{E}_1 \geq \underline{E}$ by the definition of the critical price $\hat{\pi}_1^{crit}$. As a consequence, condition (13) requires that, in the high-price state, the wage $w_1$ is set such that $E_1 = \hat{E}_1$ (since $\max[\hat{E}_1, \underline{E}] = \hat{E}_1$). In the low-price state, one of three subcases occurs: (1) $\pi_2 < \pi_2^{crit}$, (2) $\pi_2^{crit} \leq \pi_2 < \hat{\pi}_2^{crit}$, or (3) $\hat{\pi}_2^{crit} \leq \pi_2$.[27] To analyze the three subcases, we use the following results that follow directly from the definition of the critical prices $\hat{\pi}_1^{crit}$ and $\hat{\pi}_2^{crit}$ in Lemma 1:

**Corollary (to Lemma 1.1):** (i) If $\pi_1 \geq \hat{\pi}_1^{crit}$ and $\pi_2 < \hat{\pi}_2^{crit}$, $\hat{E}_1 \geq \underline{E}$, $\hat{E}_2 < \underline{E}$, and $\hat{E} = p_1 \hat{E}_1 + (1 - p_1)\underline{E} \geq \underline{E}$. (ii) If $\pi_1 \geq \hat{\pi}_1^{crit}$ and $\pi_2 \geq \hat{\pi}_2^{crit}$, $\hat{E}_1 \geq \underline{E}$, $\hat{E}_2 \geq \underline{E}$, and

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[27]$\hat{\pi}_2^{crit} > \pi_2^{crit}$ by part (iii) of Lemma 4.

$$\hat{E} = p_1 \hat{E}_1 + (1 - p_1)\hat{E}_2.$$

We now return to the three subcases defined above, and show that, in all three subcases, $E = \hat{E}$, based on the definitions of $\hat{E}$ and $E$ in expressions (4) and (8), respectively. In the paragraph above Corollary 1.1, we have already established that the incumbent's wage offer in the high-price state will be chosen so that $E_1 = \hat{E}_1$. We next show that $E = \hat{E}$ since, in the first two subcases, $\max[E_2, \hat{E}_2, \underline{E}] = \underline{E}$, while in the third subcase, $E_2 = \hat{E}_2 > \underline{E}$.

The first subcase is defined so that, by Lemma 3, the incumbent cannot profitably hire the worker at an acceptable wage in the low-price state, i.e. $\bar{E}_2 < \underline{E}$, which implies that $\max[E_2, \hat{E}_2, \underline{E}] = \underline{E}$ by expression (29). The incumbent's wage offer in the high-price state will satisfy condition (13) in the following form: $E_1 = \hat{E}_1$. Upon substituting for $E = \hat{E}$ and $\hat{E}_1$ (defined in expression (2)), the condition $E_1 = \hat{E}_1$ implies that $w_1 = \hat{w}_1$.

In the second subcase, the incumbent is the only firm in the industry that can profit from hiring the worker in the low-price state since $\pi_2 < \hat{\pi}_2^{crit}$ implies that $\hat{E}_2 < \underline{E}$ by the above-stated Corollary to Lemma 1. The incumbent will then offer the worker a wage $w_2$ which satisfies condition (13) in the following form: $E_2 = \underline{E}$. Given that $\hat{E}_2 < \underline{E}$ and $E_2 = \underline{E}$, we again obtain that $\max[E_2, \hat{E}_2, \underline{E}] = \underline{E}$, which suffices for $E = \hat{E}$ as noted below Corollary 1.1. Given that $E = \hat{E}$, the condition $E_1 = \hat{E}_1$ implies that $w_1 = \hat{w}_1$. Furthermore, $E = \hat{E}$, $E_1 = \hat{E}_1 = u(\hat{w}_1) + \delta\hat{E}$, the condition $E_2 = u(w_2) + \delta E = \underline{E}$ and expression (3) imply that $w_2$ is determined by a condition similar to condition (30) in the proof of Lemma 4 (with $w_2$ replacing $\bar{w}_2$). Therefore, $w_2 = g(w_1) = g(\hat{w}_1)$.

In the third subcase, the incumbent competes with other firms in the industry to hire the worker in the low-price state, and the wage $w_2$ will be set such that $E_2 = \hat{E}_2$ (since $\pi_2 \geq \hat{\pi}_2^{crit}$ implies that $\hat{E}_2 \geq \underline{E}$ by Corollary 1.1). $E = \hat{E}$ since the wages $w_1$ and $w_2$ will be determined by condition (13) in the form $E_z = \hat{E}_z$ for both $z = 1, 2$. Given that $E = \hat{E}$, the condition $E_z = \hat{E}_z$ implies that $w_z = \hat{w}_z$.

In all three subcases, $E = \hat{E}$ and $E_1 = \hat{E}_1$. In the first two subcases, $\max[E_2, \hat{E}_2, \underline{E}] = \underline{E}$, and, therefore, $E > \underline{E}$ iff $\hat{E}_1 > \underline{E} \Leftrightarrow w_1 > \underline{w} \Leftrightarrow \pi_1 > \hat{\pi}_1^{crit}$. In the third subcase, $E > \underline{E}$ since $E_2 = \hat{E}_2 \geq \underline{E}$ and $E_1 = \hat{E}_1 > \hat{E}_2$ (because the wage $\hat{w}_z$ increases in $\pi_z$ and $\pi_1 > \pi_2$). $\square$

## 7.4   Proof of Corollary 1.2

By Corollary 1.1, the worker's wage varies across states ($w_1 > w_2$) iff two conditions hold: $\pi_1 \geq \hat{\pi}_1^{crit} = \underline{w} + c(\alpha h)$ and $\pi_2 < \underline{w} + \bar{c}$. In the first condition, the critical value $\hat{\pi}_1^{crit}$ decreases both in the worker's human capital $h$ and the transferability of the human capital, $\alpha$, but the second condition depends neither on $h$ nor $\alpha$. The second condition, therefore, constitutes an irrelevance result: irrespective of her human capital, the worker will not be exposed to wage risk when $\pi_2 \geq \underline{w} + \bar{c}$, i.e. in case (i) of Proposition 2.[28] In case (ii) (i.e., $\pi_1 \geq \underline{w} + \bar{c} > \pi_2$), the worker will always be exposed to wage risk irrespective of her human capital (since $\hat{\pi}_1^{crit} \leq \underline{w} + \bar{c}$), but the human capital may determine the extent of the exposure. In case (iii) (i.e., $\pi_1, \pi_2 < \underline{w} + \bar{c}$), the worker will be exposed to wage risk iff the product $\alpha h$ is sufficiently high so that $\pi_1 \geq \hat{\pi}_1^{crit}$.

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[28] $\pi_2 \geq \underline{w} + \bar{c}$ implies $\pi_1 > \underline{w} + \bar{c}$ since $\pi_1 > \pi_2$. Case (i) in Proposition 2 is therefore defined as $\pi_1, \pi_2 \geq \underline{w} + \bar{c}$.

We next turn to the comparative statics of the difference $w_1 - w_2$. We focus on the cases (ii) and (iii) defined in Corollary 1.2 (since $w_1 - w_2 = 0$ in case (i)). Substituting for $w_2$ yields:

$$w_1 - w_2|_{\pi_2 < \underline{w} + \bar{c}} = w_1 - \max[g(\hat{w}_1), \hat{w}_2] = w_1 + c(\alpha h) - \max[\pi_2, \hat{\pi}_2^{crit}],$$

since $w_2 = \max[g(\hat{w}_1), \hat{w}_2] = max[\pi_2, \hat{\pi}_2^{crit}] - c(\alpha h)$ by expression (5), $\pi_2 < \underline{w} + \bar{c}$, and $\hat{\pi}_2^{crit} = g(\hat{w}_1) + c(\alpha h)$. Next, substituting for $w_1 = \hat{w}_1$ using expression (5) yields $w_1 - w_2|_{\pi_2 < \underline{w} + \bar{c}} = \min[\pi_1, \underline{w} + \bar{c}] - \max[\pi_2, \hat{\pi}_2^{crit}]$ which implies the derivative in Corollary 1.2. The sign of the derivative follows from $\partial \hat{\pi}_2^{crit} / \partial \hat{h} < 0$, as stated below Lemma 1.1. $\qquad \square$

## 7.5  Proof of Lemma 3.1

To obtain a contradiction, suppose (i) $W_1 > w_1$ and (ii) $p_1 W_1 + p_2 W_2 \leq p_1 w_1 + p_2 w_2$. Let $\Delta W_1 = W_1 - w_1$ and $\Delta W_2 = W_2 - w_2$. Then:

$$
\begin{aligned}
E^L &= \frac{1}{1-\delta}(p_1 u(W_1) + p_2 u(W_2)) \\
&= \frac{1}{1-\delta}(p_1 u(w_1 + \Delta W_1) + p_2 u(w_2 + \Delta W_2)) \\
&< \frac{1}{1-\delta}(p_1(u(w_1) + u'(w_1)\Delta W_1) + p_2(u(w_2) + u'(w_2)\Delta W_2))) \quad (31) \\
&= E + \frac{1}{1-\delta}(p_1 u'(w_1)\Delta W_1 + p_2 u'(w_2)\Delta W_2)) \\
&\leq E \quad (32)
\end{aligned}
$$

The inequality in equation (31) follows from $u''(\dot{)} < 0$ and condition (i), and the inequality in equation (32) is obtained by using condition (ii). Thus, when $W_1 > w_1$ long-term contracting cannot reduce the expected wage bill relative to short-term contracting while at the same time satisfying the worker's PC (21) constraint. Also, the worker's PC (21) cannot be satisfied if both $W_1 \leq w_1$ and $W_2 < w_2$. We can therefore conclude that long-term contracting can only reduce the expected wage bill while satisfying the PC (21) if $W_1 \leq w_1$ and $W_2 \geq w_2$.

A similar proof by contradiction shows that $W_1 \geq W_2$. Finally, if $W_1 < w_1$, then the worker's incentive compatibility constraint can only be satisfied if $W_2 > w_2$. $\qquad \square$

## 7.6  Proof of Lemma 3.2

First note that $R \leq R^L$ since the incumbent's rents from employing the worker under a long-term contract are at least as high as those under the short-term contract (which is a feasible long-term contract). Therefore, $\bar{W}_z \geq \bar{w}_z$. As a result, the constraint $ICI_z$ (equation (23)), can only bind in state $z$ when $W_z \geq w_z$.

We next turn to constraint $ICW_z$ (equation (22)). If $W_z < w_z$ in any state $z$, the constraint $ICW_z$ implies that the worker's PC (21) holds, and also that the constraint $ICW_{z'}$ holds, where $z'$ denotes the other state.[29]

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[29]To see this, note that the PC (21) could not hold if $W_z < w_z$ and $W_{z'} \leq w_{z'}$. Therefore, $W_z < w_z$

It remains to show that constraint $ICW_1$ will bind if $W_1 < w_1$ and that constraint $ICW_2$ will bind if $W_2 < w_2$. We start with the first result. To obtain a contradiction, suppose that a contract $(W_1, W_2)$ is the solution to the optimization problem, $W_1 < w_1$ and the constraint $ICW_1$ does not bind. In this case, since $u()$ and $E^L$ are continuous, there must exist a $\delta > 0$ for which $u(W_1 - \delta) + \delta E^L > \max[\hat{E}_1, \underline{E}]$, so that the constraint $ICW_1$ still holds. Moreover, all other constraints will also continue to hold after the wage $W_1$ is reduced to $W_1 - \delta$.[30] However, $p_1(W_1 - \delta) + p_2 W_2 < p_1 W_1 + p_2 W_2$ and reducing the wage $W_1$ did not increase the incumbent's expected financing cost, so that the contract $(W_1, W_2)$ was not a solution to the optimization problem.

We next show that the constraint will $ICW_2$ will bind if $W_2 < w_2$ and $\phi'(W_2 + c(h) - \pi_2) = f > 0$ since $\pi_2 < W_2 + c(h)$. To obtain a contradiction, suppose that a contract $(W_1, W_2)$ is the solution to the optimization problem, $W_2 < w_2$ and the constraint $ICW_2$ does not bind. Then, there must exist a $\delta > 0$ for which $u(W_2 - \delta) + \delta E^L > \max[\hat{E}_2, \underline{E}]$, so that the constraint $ICW_2$ sill holds. Moreover, all other constraints continue to hold after the wage $W_2$ is reduced to $W_2 - \delta$.[31] However, reducing the wage $W_2$ did reduce the incumbent's expected financing cost, so that the contract $(W_1, W_2)$ was not a solution to the optimization problem. $\qquad\square$

## 7.7 Proof of Proposition 2

Given Assumption 2, it remains to consider wages $W_2 \geq w_2$ and $W_1 \leq w_1$ for which the incumbent may have to finance a loss in the low-price state. The expected financing cost is therefore given by $F^L(W_1, W_2) = p_2 \phi(W_2 + c(h) - \pi_2)$ where $\phi$ is the financing cost function (10).

By part (ii) of Lemma 5, the objective function $Z(W_1, W_2)$ in expression (20) will have to be maximized subject to the binding constraint $ICW_1$, i.e. constraint (22) with $z = 1$, which we now denote as

$$B(W_1, W_2) := u(W_1) + E^L = u(W_1) + \frac{\delta}{1-\delta}(p_1 u(W_1) + p_2 u(W_2)) - E_1 = 0,$$

where we have substituted for $E^L = (p_1 u(W_1) + p_2 u(W_2))/(1 - \delta)$ and have used that $\max[\hat{E}_1, \underline{E}] = E_1$ by equation (12).

Setting up the Lagrangian for maximizing $Z(W_1, W_2)$ subject to the binding constraint $ICW_1$ yields the conditions:

$$B(W_1, W_2) = 0 \text{ and } \lambda = \frac{Z_1}{B_1} = \frac{Z_2}{B_2},$$

where $\lambda$ denotes the Lagrange multiplier and subscripts denote derivatives of the functions $Z$ and $B$. The first condition is the first condition stated in Proposition 2. The second

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requires $W_{z'} > w_{z'}$, but $W_{z'} > w_{z'}$ and the PC (21) imply that the constraint $ICW_{z'}$ holds.

[30] The constraint (23) will continue to hold for both $z = 1, 2$ since $W_1$ was reduced to $W_1 - \delta$ while $W_2$ remained unchanged. The constraint (21) will continue to hold since at the new wage $W_1 - \delta < w_1$, the constraint $ICW_1$ can only hold if $E^L > E$. The constraint $ICW_2$ will hold since we did not change $W_2$.

[31] The constraint (23) will continue to hold for both $z = 1, 2$ since $W_2$ was reduced to $W_2 - \delta$ while $W_1$ remained unchanged. The constraint (21) will continue to hold since at the new wage $W_2 - \delta < w_2$, the constraint $ICW_2$ can only hold if $E^L > E$. The constraint $ICW_1$ will hold since we did not change $W_1$.

equation in the second condition requires:

$$\frac{Z_1}{B_1} = \frac{p_1}{u'(W_1)(1 - \delta + \delta p_1)/(1 - \delta)} = \frac{p_2(1 + \phi'(W_2 + c(h) - \pi_2))}{u'(W_2)\delta p_2/(1 - \delta)} = \frac{Z_2}{B_2}.$$

Rearranging the equation in the middle yields the second condition stated in Proposition 2.

The conditions identify a minimum if

$$(Z_{11} - \lambda B_{11})dW_1^2 + 2(z_{12} - \lambda B_{12})dW_1 dW_2 + (Z_{22} - \lambda B_{22})dW_2^2 \geq 0,$$

where subscripts denote second derivatives, and $Z_{11} = Z_{12} = B_{12} = 0$. Substituting for the remaining second derivatives yields:

$$-\lambda u''(W_1)\frac{1 - \delta + \delta p_1}{1 - \delta}dW_1^2 + (p_2\phi''(W_2 + c(h) - \pi_2) - \lambda u''(W_2)\frac{p_2\delta}{1 - \delta}dW_2^2 \geq 0.$$

Given that $u''(\cdot) \leq 0$, a sufficient condition is $\phi''(W_2) \geq 0$, which will be true for the financing cost function (10).

# Figure 1
## Proposition 1
## Wages as a function of prices in different states

The figure depicts the different wages that the worker will earn as a function of industry prices in different states. The x-axis shows output price in state 1 while the y-axis shows output price in state 2. Without loss of generality we assume $\pi_1 \geq \pi_2$, so the relevant regions lie below the 45-degree line. The relevant areas depend on critical prices $\hat{\pi}_1^{crit}$, $\hat{\pi}_2^{crit}$, $\pi_1^{crit}$, and $\pi_2^{crit}$. $\hat{\pi}_z^{crit}$ are the minimum prices under which other firms in the industry besides the incumbent are willing to make an offer to the worker. $\pi_z^{crit}$ are the minimum prices under which the incumbent is willing to make an offer to the worker. $\pi_2^{crit}$ and $\hat{\pi}_2^{crit}$ are descending and non-linear when $\pi_1 > \hat{\pi}_1^{crit}$ because in this area the worker is willing to accept a lower wage in state 2 to maintain her human capital and earn from it in state 1.
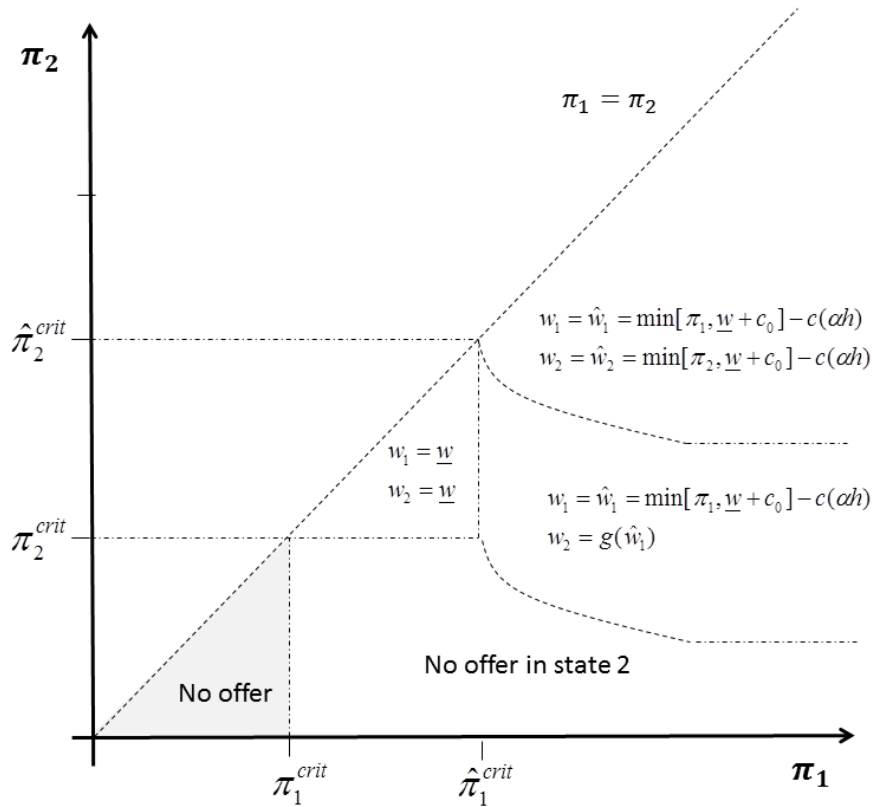


$$w_1 = \hat{w}_1 = \min[\pi_1, \underline{w} + c_0] - c(\alpha h)$$
$$w_2 = \hat{w}_2 = \min[\pi_2, \underline{w} + c_0] - c(\alpha h)$$

$$w_1 = \underline{w}$$
$$w_2 = \underline{w}$$

$$w_1 = \hat{w}_1 = \min[\pi_1, \underline{w} + c_0] - c(\alpha h)$$
$$w_2 = g(\hat{w}_1)$$

No offer in state 2

No offer

$\pi_1 = \pi_2$

$\hat{\pi}_2^{crit}$

$\pi_2^{crit}$

$\pi_1^{crit}$

$\hat{\pi}_1^{crit}$

$\pi_2$

$\pi_1$

# Figure 2
# Proposition 2
# Optimal long-term contract with full employment commitment

The figure depicts the optimal long-term wages $(W_1^*, W_2^*)$ with full employment commitment as a function of the short-term contract $(w_1, w_2)$, for the case without financing costs. The x-axis shows the worker's wage in state 1 while the y-axis shows the wage in state 2. The optimal long-term contract wage, as well as the short-term contract wage, lies on the graphical representation of the worker's incentive compatibility constraint in state 1 $(ICW_1)$, as we show this is the only binding constraint. The optimal long-term contract wage is determined by the point where the worker's $ICW_1$ is parallel with the incumbent's constant expected present value of wage payments. At this point the wage bill cannot be minimized further while ensuring the worker wants to participate. The figure shows the case with no financing costs; when financing costs $f > 0$ the constant expected value of wage payments need to include financing costs as well. These are no longer a straigh line, but the geometric description of the optimal contract remains the same.
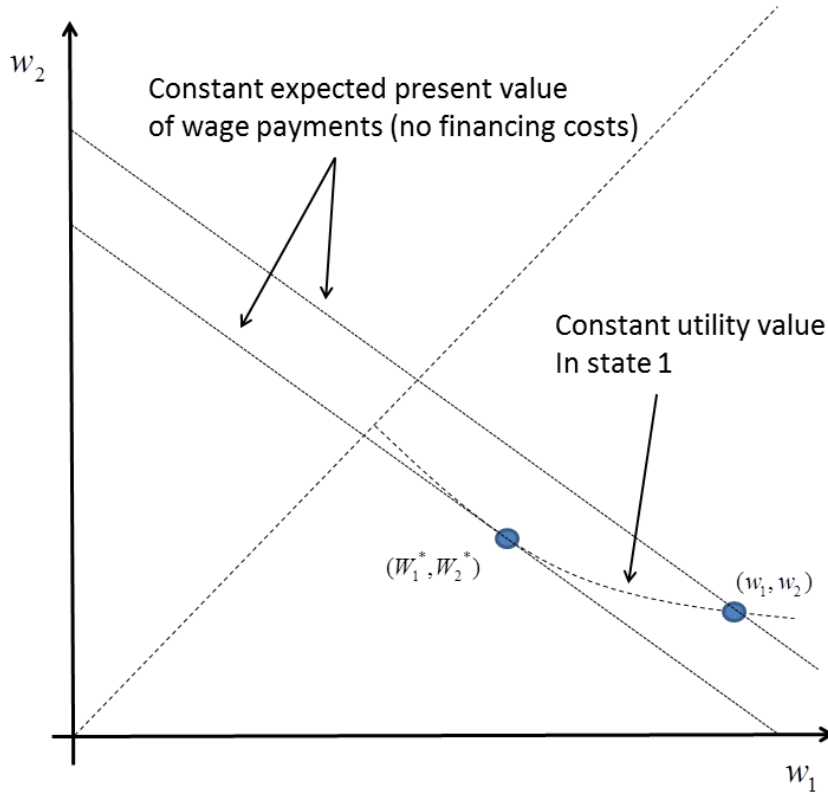
## Figure 3
## Proposition 3
## Optimal long-term contract with limited employment commitment

The figure depicts the optimal long-term wages $(W_1^*, W_2*)$ with limited employment commitment as a function of the short-term contract $(w_1, w_2)$, for the case without financing costs. The x-axis shows the worker's wage in state 1 while the y-axis shows the wage in state 2. As in Figure 2, the optimal long-term contract wage, as well as the short-term contract wage, lies on the graphical representation of the worker's incentive compatibility constraint in state 1 $(ICW_1)$, as we show this is still a binding constraint. Unlike Figure 2, however, the optimal long-term contract is now constrained by the incumbent's willingness to participate in state 2 $(ICI_2)$. The incumbent's participation constraint depends on the rents earned, which includes rents from providing insurance, and thus moves as $W_z$ changes. The optimal long-term contract can be found where both constraints $(ICI_2$ and $ICW_1)$ bind. When financing costs $f > 0$ when financing costs $f > 0$ the constant expected value of wage payments need to include financing costs as well. These are no longer a straigh line, but the geometric description of the optimal contract remains the same.